DOUBLE VECTOR-INTEGRAL-EQUATION METHOD FOR MICROSTRIP-LINES ON PHOTONIC BAND-GAP SUBSTRATES

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Abstract

This paper presents a double vector-integral-equation (DOVIE) method for the modeling of integrated circuit components on artificial (photonic band-gap) substrates. The proposed computational method deals with the interaction of circuit components (continuous plane wave spectrum) and artificial materials (discrete plane wave spectrum, Floquet modes). The method of solution involves two stages of vector integral-equations. The first integral equation formulation is to find the Green's function for a PBG structure. A spectral-domain moment-method is applied to the second vector integral-equation to determine the fields or currents on the circuit components and the associated parameters of interest. Field solutions of microstrip lines on PBG substrates are discussed. The results of this work initiate research for many innovated microwave and millimeter-wave integrated components and devices.

I. Introduction

In recent years, with the advances of material processing technology, there has been growing importance in the development of advanced artificial materials. For examples, photonic crystals, which are artificial materials made of two or three dimensional periodic dielectrics, are in analogy to crystals made of periodic atoms or molecules that exhibit electron band gaps [1]. In photonic band gap (PBG) materials, the periodic implants are comparable to a wavelength in size and may be metallic, magneto-dielectric, ferromagnetic, ferroelectric, or active, where the band gap can be controlled with external current or voltage biases or light sources. Many technologies may benefit from those artificial materials, where electromagnetic wave propagation is properly controlled. Recently, there is increasing interest in microwave and millimeter-wave applications of PBG materials [2-5]. The alternation of object signature in an identifiable way, broadband absorber, and narrow-band frequency selective surfaces are of many important applications. Artificial material properties are scaleable and applicable to a wide range of frequencies. Materials can be constructed for a given geometry with millimeter dimensions for microwave control and with micron dimensions for infrared control. Electromagnetic wave theory and computational techniques are necessary to determine the fundamental physical principles of material properties and the design of devices and components from microwave to optical frequencies. The existing analyses including plane-wave expansion method, integral equation method, finite difference method, and finite element method, are limited to either periodic structures or defects with highly localized modes. Periodic structures with anomalies are important in many areas of engineering and science. Undesired radar cross section (RCS) in frequency selective surfaces, radiation degradation in phased array, trapped-wave modes in periodic waveguides, and x-ray diffraction from crystals with defects are some of the examples. The implementations of PBG materials into integrated circuit and antenna structures will result in many new technologies.

In the current knowledge of field, there is no appropriate computational scheme for the field solution of integrated circuit component interaction with PBG materials. In this work, we propose a double vector-integral-equation (DOVIE) method. This DOVIE method works for general periodic structures with anomalies. In the modeling process, there are two vector integral-equations to be solved systematically and sequentially. The first integral equation formulation is to find the electromagnetic fields in an artificial material structure with infinite phased arrays of δ sources. A continuous phase array method transforms the fields due to infinite phased arrays to those due to a single δ source. Those fields after transformation are the dyadic Green's function for the second vector integral equation that is solved to determine the parameters of interest. This paper presents our investigation of the characteristics of microstrip transmission lines on PBG substrates. The results are validated with limiting cases and an effective uniform medium approach.

2. DOVIE Method for a Microstrip Line on a PBG Substrate

The geometry of a microstrip line on a PBG substrate is shown in Figure 1. The substrate, assumed infinitely large, is a dielectric material with planar periodic blocks. The metal strip is aligned with the arrays of material blocks. For simplicity, it is also assumed that the microstrip is narrow and only the

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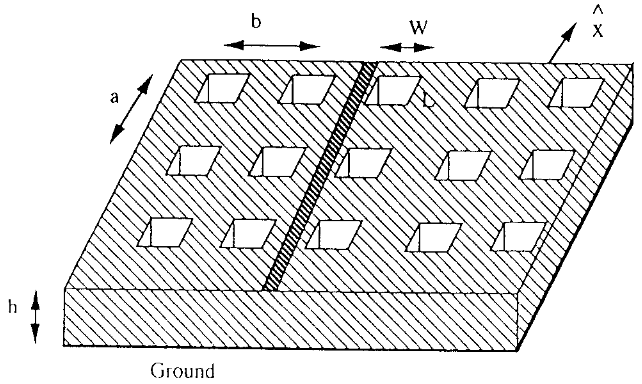

Figure 1. A microstrip line on a PBG substrate.

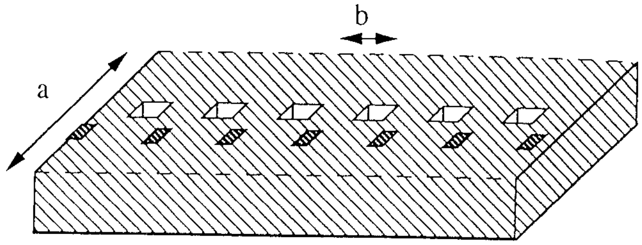

Figure 3. An infinite array of microstrip segment with progressive phase shift ϕ_y.

longitudinal current exist. Conventional numerical methods such as the method of moments, finite element method, and finite difference method can not deal with such problem involving the interaction of circuit elements (continuous plane-wave spectrum) and periodic structures (discrete plane-wave spectrum). For the convenience of discussion, it is assumed that the current is uniform across the strip, which is a reasonable assumption for a conventional microstrip line. Extension to more sophisticate basis functions can be incorporated with a straightforward modification. In the moment method analysis, the microstrip longitudinal current in Figure 1 is expanded in terms of basis function within a unit cell ($0 \le x \le a$) as was shown in Figure 2. The microstrip current has a phase constant β_x and is a periodic function of x. In order to apply the Galerkin's procedure in the moment method, we need to evaluate the reaction of the current basis functions. Therefore, it is necessary to compute the $\hat{x}$ electric field component E_x at the air/material interface due to a current basis function. The inner products of E_x and the basis functions (testing functions) will be matrix elements of the characteristic matrix. The deterministic equation is found by setting the matrix determinant to zero. The propagation constant β_x is the root of the deterministic equation.

Instead of solving the problem directly, we first consider the geometry shown in Figure 3, where there are infinite phased arrays of current segments with progressive phase shifts ϕ_y. With the use of Floquet (or Bloch) theorem and periodic boundary conditions, the problem is simplified to the modeling of electromagnetic waves within a unit cell. This problem can be solved with a 3-D integral equation formulation and the method of moments for periodic structures. The details are described in [6-7].

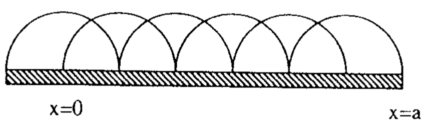

Figure 2. Piecewise sinusoidal basis functions along the micrstrip line in a unit cell.

A material block is at the center of the cell with length L (along the $\hat{x}$ axis), width W (along the $\hat{y}$ axis), and the thickness T (along the $\hat{z}$ axis). The substrate with thickness Δ is the distance measured from the bottom of the block to the layer interface. The boundary-value problem shown in Figure 3 is formulated through an electric volume integral equation. The electric fields within the material blocks are the displacement currents. The fields in the structure are due to a basis function $p_i(x, y)$. In the moment method procedure, these displacement currents are discretized into many small cells within which the fields are assumed constants, but with unknown coefficients. For planar periodic structures, we may express the components of the dyadic Green's function for a homogeneous substrate in terms of Floquet modes (plane wave expansion) as

$$G_{uv} = \frac{1}{ab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \tilde{G}_{uv}\, e^{-jk_x(x-x')-jk_y(y-y')}, \qquad (1)$$

where $k_x = \frac{2m\pi}{a} + \beta_x$ and $k_y = \frac{2n\pi}{b} + \frac{\phi_y}{b}$. u or v is either x, y, or z. β_x is the propagation constant of the transmission line. $\tilde{G}_{uv}$ is the spectral Green's function component and is a function of spectral variables k_x and k_y, z, z', and the material parameters. The solutions of the electric field in the $\hat{x}$ direction for the phased array problem (Figure 3) can be written in the following form:

$$E_x^i(\beta_x, \phi_y) = \frac{1}{ab} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \tilde{E}_x^i(m, n, \beta_x, \phi_y) e^{jk_x x + jk_y y}, \qquad (2)$$

The solution of the electric field due to a microstrip current segment only can be found from Eq. (2) through

$$E_x^{ir}(x, y, \beta_x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} E_x^i \, d\phi_y \qquad . \qquad (3)$$

The integration in (3) is the superposition of the solutions to the periodic phased arrays. The key is the superposition principle and phase cancellation. When we perform the integration in (3), we are in effect canceling out all other line current sources in the infinite arrays except the one with zero phase angle.

With the result in (3), we may proceed the Galerkin's procedure to find the characteristics of a microstrip line. The matrix elements of the characteristic matrix are

$$A_{ii'} = \frac{1}{2\pi} \int_{-\pi}^{\pi} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \tilde{E}_x^i(m,n,\beta_x,\phi_y) q_{i'}(k_x,k_y) d\phi_y, \qquad (4)$$

where

$$q_{i'}(k_x,k_y) = \iint p_{i'}(x,y) e^{jk_x x + jk_y y} dxdy \qquad (5)$$

and i and i' are the indices for the basis functions on the strip. The eigenvalues (propagation constants) β_x are obtained from the roots of the characteristic equation

$$\det[A] = 0 \qquad . \qquad (6)$$

For a lossless structure, the propagation constant of the guided wave is a real number, and a bisection method for finding the roots of nonlinear functions is used. The characteristic impedance of the line is defined based on a power-current formula. Direct evaluation of Poynting vector is difficult, due to the discontinuity of electric fields at the edge of the periodic blocks. A more applaudable approach is to evaluate the magnetic field stored within a unit cell ($0 \le x \le a$) and use the formula [8]

$$P_{ave} = \frac{2}{a(d\beta_x / d\omega)} W_m \quad , \qquad (7)$$

where

$$W_m = \frac{\mu_0 a}{8\pi} \int_0^{\infty} \int_{-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \left| \tilde{H}(k_x,k_y,z) \right|^2 dk_y dz \qquad (8)$$

and $\tilde{H}(k_x,k_y,z)$ is the magnetic field for a Floquet mode (see Equation 2).

III. Results and Discussions

For conventional artificial dielectrics, the periodic elements are much smaller than a wavelength and the electromagnetic properties of the materials can be characterized accurately by an effective dielectric constant. For the present investigation of the PBG materials, the periodic elements are comparable to a wavelength and propagation band gaps exist in such structures. The validity of the analysis is first checked against the case where the implanted blocks are as large as the unit cell. The substrate parameters are $h = 1$ mm and $\varepsilon = 4$. The implanted material-blocks have the following parameters: $L = 3$mm, $W = 3$mm, $T = 0.5$mm, $\Delta = 0$, and $\varepsilon_e = 10$. The periods of the arrays are the same as the block dimensions ($a = b = 3$ mm) so that the implanted material blocks fill the lower half of the substrate. The geometry is essentially a two-layer structure with top layer $\varepsilon = 4$ and the bottom layer $\varepsilon_e = 10$. Each layer has thickness 0.5 mm. The SDA solutions for such a structure are known [9]. The results of this test case are shown in Tables I and II for phase constant method, the periodic blocks are divided into 9 rectangular boxes where the electric fields are treated as displacement currents found from a moment method procedure. It is seen that the present DOVIE method agrees very well with the SDA.

The results of the propagation constant and the characteristic impedance of a microstrip on a PBG substrate are shown in Figures 4 and 5, respectively. For comparison, the SDA results for an effective uniform substrate are also shown. In numerical computation, 1681 Floquet modes with $M_x = M_y = M_z = 3$ are used. Four sections of 16-point Gaussian integration are used for the integral in (4). Generally, the spacing between the periodic blocks along the microstrip direction affects significantly the guided wave characteristics. There are two interesting features of the microstrip-line mode that are very different from a conventional microstrip-line mode. When $\beta_x a \approx \pi$, there exists a mode gap within which the propagation mode vanishes. Also, when $\beta_x \ge 2\pi / a - k_0$, the guided wave modes become leaky-waves which are fast waves with complex propagation constants. It is interesting to see that at low frequencies the artificial substrate is like an effective uniform medium. As frequency increases to near the band-gap zone, the line impedance increases drastically and the microstrip line becomes open-circuited at the band-gap edge. When the frequency increases further and moves just out of the band-gap zone, the microstrip line is like a short circuit. The emphasis of this paper is on the numerical method. Further investigations of the microstrip-line characteristics including the effects of the strip location, the dimensions of the periodic elements, and periods will be discussed elsewhere.

Table I
Validity Check of Phase Constant β/k_0
With the Case of A Two-Layer Uniform Substrate

Frequency (GHz)	SDA	DOVIE Method
2	1.885	1.877
4	1.895	1.887
6	1.908	1.899
8	1.923	1.914
10	1.939	1.930
12	1.957	1.947
14	1.975	1.965
16	1.994	1.984
18	2.013	2.003
20	2.033	2.023

Table II
Validity Check of Characteristic Impedance
With the Case of A Two-Layer Uniform Substrate

Frequency (GHz)	SDA	DOVIE Method
2	70.6	71.2
4	71.0	71.5
6	71.3	71.4
8	72.1	71.7
10	72.6	73.0
12	74.5	74.2
14	75.6	75.4
16	77.3	77.4
18	78.8	79.6

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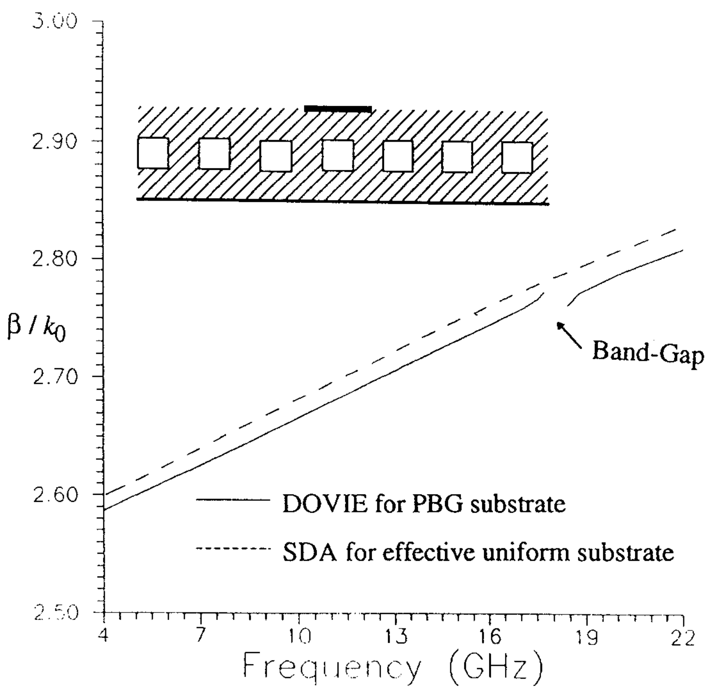

Figure 4. Propagation constant of a microstrip line on a PBG substrate. $h = 1$ mm, $\varepsilon = 10$, $L = 0.5$ mm, $W = 0.5$ mm, $T = 0.4$ mm, $\Delta = 0.3$ mm, and $\varepsilon_e = 2$, $a = b = 3$ mm, and the strip width $w = 1$ mm.

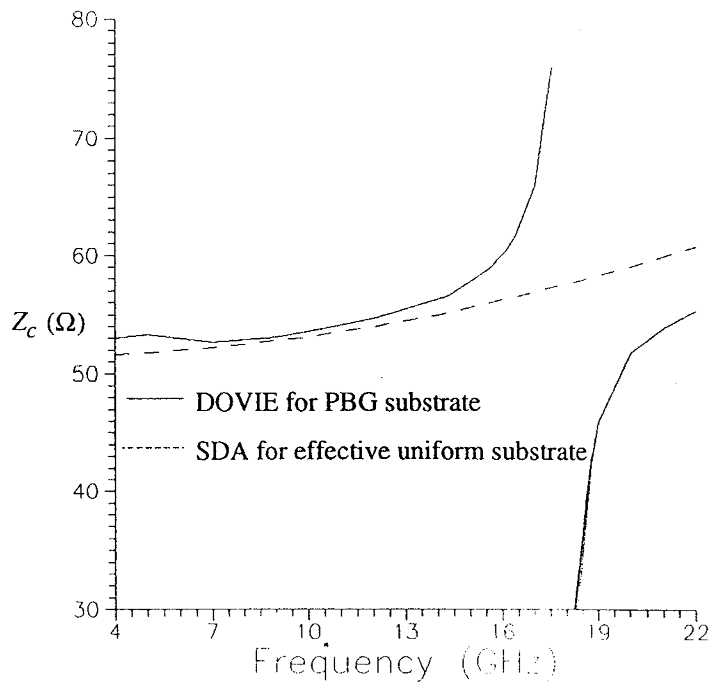

Figure 5. Characteristic impedance of a microstrip line on a PBG substrate. $h = 1$ mm, $\varepsilon = 10$, $L = 0.5$ mm, $W = 0.5$ mm, $T = 0.4$ mm, $\Delta = 0.3$ mm, and $\varepsilon_e = 2$, $a = b = 3$ mm. and the strip width $w = 1$ mm.